

## Problem 9

(III) For small angles  $\theta$ , the numerical value of  $\sin \theta$  is approximately the same as the numerical value of  $\tan \theta$ . Find the largest angle for which sine and tangent agree to within two significant figures.

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### Solution

Evaluate  $\sin \theta$  and  $\tan \theta$  for several values of  $\theta$ .

$\theta$	$\sin \theta$	$\tan \theta$
0.19	0.188859	0.19232
0.20	0.198669	0.20271
0.21	0.20846	0.213142
0.22	0.21823	0.223619
0.23	0.227978	0.234143
0.24	0.237703	0.244717
0.25	0.247404	0.255342

Notice that once  $\theta$  gets to 0.25 radians,  $\sin \theta$  rounds to 0.25, whereas  $\tan \theta$  rounds to 0.26. We want the value of  $\theta$  that gives  $\tan \theta = 0.245$ , that is,

$$\theta = \tan^{-1}(0.245) \approx 0.240267 \text{ rad.}$$

For this value of  $\theta$ ,

$$\sin \theta \approx 0.237962$$

$$\tan \theta = 0.245$$

$\sin \theta$  rounds to 0.24, and  $\tan \theta$  just barely rounds to 0.25. This is the maximum value of  $\theta$  for which  $\sin \theta$  and  $\tan \theta$  agree to within two significant figures.